

# Optimal Design of Measurement-Type Current Transformer Using Shuffled Frog Leaping Algorithm

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**Abstract**—In this paper a new approach based on Shuffled Frog Leaping Algorithm (SFL) for measurement-type current transformer (CT) design has been presented. This algorithm can present designed parameters of sample current transformer so that minimizes ratio and phase displacement errors to 1.2 times of rated current and transformer made cost also. Finally, several current transformers with different rated values are designed and results show that the proposed approach can be used for optimal design of current transformer perfect.

**Keywords**—Current Transformer; Shuffled Frog Leaping Algorithm; phase displacement error; ratio error

## I. INTRODUCTION

The current transformers (CT) can be classified by their usage into two types: one for measuring the working current, the other for measuring the fault current to provide control signal to the protective devices of power system. Generally, the current measured by the first type is not greater than the rated current, and its main purpose is to obtain the effective value of current. While the current measured by the second type is mainly the short circuit current that may be 10 times greater than the rated current [1].

Shuffled frog leaping (SFL) is a population based, cooperative search metaphor inspired by natural memetics. Its ability of adapting to dynamic environment makes SFL become one of the most important memetic algorithms. In order to improve the algorithm's stability and the ability to search the global optimum, a novel 'cognition component' is introduced to enhance the effectiveness of the SFL, namely frog not only adjust its position according to the best individual within the memplex or the global best of population but also according to thinking of the frog itself. According to the simulation results, adding the cognitive behavior to SFL significantly enhances the performance of SFL in solving the optimization problems, and the improvements are more evident with the scale of the problem increasing.

In this paper, in addition to precise investigation of performance and construction of current transformer, a method based on Shuffled Frog Leaping Algorithm has been presented for optimal design of current transformer. The presented algorithm, with minimizing the objective function in which ratio and phase displacement errors and construction cost has been regarded, is capable of designing a transformer which has optimal construction cost besides minimum error.

## II. CURRENT TRANSFORMER PERFORMANCE

Equivalent circuit of current transformer is shown in Fig. 1.  $R_2$  is resistance of secondary winding,  $I_e$  is magnetizing current, and  $R_b$  and  $X_b$  are resistance and reactance of burden respectively. Phase diagram of current transformer is shown in Fig. 2. Ratio error is difference between amplitude of  $I_1, I_2$  and  $\theta$  is phase displacement error.

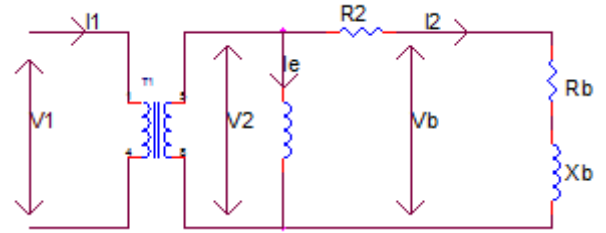


Figure 1. Simplified equivalent circuit of current transformer

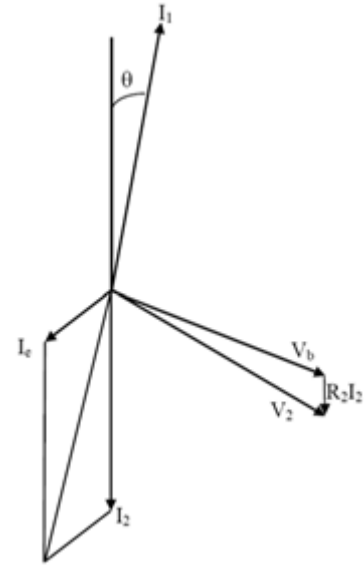


Figure 2. Current transformer phase diagram

With respect to equivalent circuit in Fig. 1, Equations (1)-(6) have been resulted. With considering that core loss is negligible, in this transformer, equivalent resistance of core is not taken into account so that we can substitute  $M_r$  with a large value.

$$I_1 = \left[ \frac{1}{R_m} + \frac{1}{(R_2 + R_b) + j(X_2 + X_b)} \right] \times V_2 - jI_m \quad (1)$$

$$A + jB = \left[ \frac{1}{R_m} + \frac{1}{(R_2 + R_b) + j(X_2 + X_b)} \right] \quad (2)$$

$$I_1 = [A + jB] \times V - jI_m \quad (3)$$

$$|I_1|^2 = A^2 V_2^2 + B^2 V_2^2 - 2I_m B V_2 + I_m^2 \quad (4)$$

$$(A^2 + B^2)V_2^2 - 2I_m B V_2 + B(I_m^2 - |I_1|^2) = 0 \quad (5)$$

$$I_2 = \left[ \frac{1}{(R_2 + R_b) + j(X_2 + X_b)} \right] \times V_2 \quad (6)$$

Equations (1)-(6) are in per-unit system and among them (1), (5) and (6) are more important.

### III. CURRENT TRANSFORMER CONSTRUCTION

The ring-type current transformer as shown in Fig. 3 is considered for design procedure, primary winding of this transformer is composed of one turn and actually is current carrying conductor passes through the CT.

Wire material is copper. Various cores are used in current transformers, the most common cores are F, P&R and K which their characteristics and magnetization curves are shown in Table I and Fig. 4.

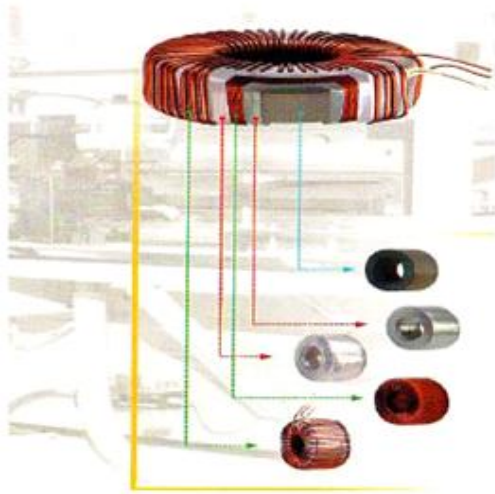


Figure 3. Construction of ring-type current transformer

TABLE I.  
COEFFICIENTS OF CORES IN EFFECTIVE PERMEABILITY EQUATION

Material	F	P&R	K
a	$10^7$	$4.49 \times 10^6$	$2.35 \times 10^6$
b	$6.66 \times 10^5$	$6.32 \times 10^7$	$6.54 \times 10^4$
c	-7200	$-3.73 \times 10^5$	-686
d	0.648	3.48	-0.319
e	3.00	37.7	0.511
f	0	4.01	0

$$\mu_e = ((a + bH + cH^2)/(1 + dH + eH^2 + fH^3))^{1/2} \quad H \text{ in Oersteds}$$

### IV. OBJECTIVE FUNCTION

Objective function for designing problem can be expressed as follows:

$$fitness = 1 / ((k_1 Sumdis + k_2 Sumph + k_3 ((T\_Price_{ref} - T\_Price)^2) + k_4 Ploss + k_5 burden\_sq)) \quad (7)$$

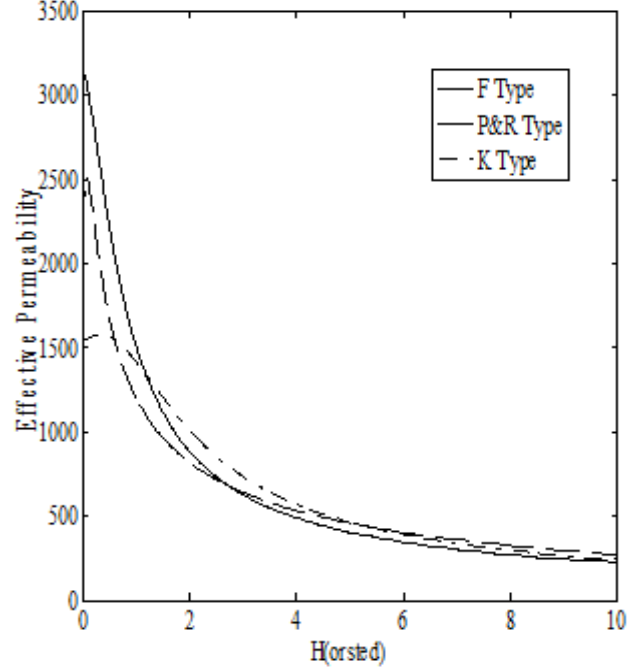


Figure 4. Effective magnetic permeability in terms of magnetic field intensity for three different cores

$$Sumdis = \sum_{i=1}^5 [(|I_{1i}| - |I_{2i}|)^2] \quad (8)$$

$$Sumph = \sum_{i=1}^5 [(\angle I_{1i} - \angle I_{2i})^2] \quad (9)$$

$$T\_Price = Co\_Price + Cu\_Price \quad (10)$$

$$Ploss = R_2 I_2^2 \quad (11)$$

$$Z_{bnew} = \sqrt{(R_b^2 + X_t^2)} \quad (12)$$

$$burden\_sq = (1 - Z_{bnew})^2 \quad (13)$$

Where:

*Sumdis* : sum of ratio errors squares

*Sumph* : sum of phase displacement errors squares  
(Phase displacement errors are in terms of minute)

*Co \_ Price* : Core cost (\$)

*Cu \_ Price* : Copper cost (\$)

*T \_ Price* : Total cost (\$)

*T \_ Price<sub>ref</sub>* : Reference total cost (\$)

*Ploss* : Cupper losses in watts

*Z<sub>bnew</sub>* : Burden impedance

*burden\_sq* : Square of burden impedance error

Ratio and phase displacement errors have been calculated in 0.25, 0.5, 0.75, 1 and 1.2 times of rated current and inserted in objective function. cross section of core and wire have been calculated in rated current, then by using this cross sections and magnetization curve of core, the errors have been calculated in mentioned coefficients of rated current. According to the presented objective function, it is clear that minimizing of objective function, presents design parameters of transformer with minimum construction cost besides minimum errors.

## V. SHUFFLED FROG LEAPING ALGORITHM

SFL, a new member in the family of memetic algorithms, is a population based, cooperative search metaphor inspired by natural memetics. It is originated from the research of food hunting behaviors of frog. Researchers found that, in theory at least, individual members of the school can profile from the discoveries and previous experience of all other members of the school during the search for food. The advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches. Their behaviors are unpredictable but always consistent as a whole, with individuals keeping the most suitable distance. Through the research of the behaviors of similar biological communities, it is found that there exists a social information sharing mechanism in biological communities. This mechanism provides an advantage for the evolution of biological communities, and the basis for the formation of SFL [7, 8].

The algorithm uses memetic evolution in the form of infection of ideas from one individual to another in a local search. A shuffling strategy allows for the exchange of information between local searches to move toward a global optimum. In essence, combines the benefits of the genetic based memetic algorithm (MA) s and the social behavior-based particle swarm optimization (PSO) algorithms [9]. SFL algorithm, originally by Eusuff and Lansey in 2003, Likes GA, PSO, is developed an optimization algorithm based on population, can be used to solve many complex optimization problems, which are nonlinear, non-differentiable and multi-modal. The most prominent merit of SFL is its fast convergence speed [10]. However, in the original SFL algorithm, every frog update its position according to the best solution, because of the influence of the local best solution, every frog will constringe about the local best solution quickly. In order to improve the algorithm's stability and the ability to search the global optimum, a novel 'cognition component' is introduced, namely frog not only adjust its position according to the best individual within the memeplex or the global best of population but also according to thinking of the frog itself.

## VI. OVERVIEW SHUFFLED FROG LEAPING ALGORITHM

The SFL Algorithm is a memetic meta-heuristic that is designed to seek a global optimal solution by performing an

informed heuristic search using a heuristic function. It is based on evolution of memes carried by interactive individuals and a global exchange of information among the population.

The SFL algorithm progresses by transforming "frogs" in a memetic evolution. In this algorithm, frogs are seen as hosts for memes and described as a memetic vector. Each meme consists of a number of memo types. The memo types represent an idea in a manner similar to a gene representing a trait in a chromosome in a genetic algorithm. The SFL does not change the physical characteristics of an individual rather it progressively improves the ideas held by each frog in a so called virtual population.

The frogs can communicate with each other, and can improve their memes by infecting (passing information) each other. Improvement of memes results in changing an individual frog's position by adjusting its leaping step size. Based on this abstract model of virtual frogs, the SFL algorithm draws on PSO as a local search tool and the idea of competitiveness and mixing information from parallel local searches to move toward a global solution from the Shuffled complex evolution (SCE) algorithm [11].

The sample of virtual frogs constitutes a population. The population is partitioned into subsets described as memeplexes. The memeplexes can be perceived as a set of parallel frog cultures attempting to reach some goal. Each frog culture proceeds towards their goal exchanging ideas independently in parallel. Frog leaping improves an individual's meme and enhances its performance towards the goal. Within each memeplex, the individual frogs hold information can be infected by other's ideas, and hence they experience a memetic evolution. After a defined number of memetic evolution steps, information is passed between memeplexes in a shuffling process. Shuffling enhances the meme quality after being infected by the frogs from different memeplexes, ensures that the cultural evolution towards anyParticular interest is free from bias. After this, this local search and shuffling process continues until defined convergence criteria are satisfied. The SFL algorithm is a combination of deterministic and random approaches. The deterministic strategy allows the algorithm to use response surface information effectively to guide the heuristic search. The random elements ensure the flexibility and robustness of the search pattern. The SFL algorithm starts with an initial population of "q" frogs created randomly within the feasible space "Q". For D-dimensional problems, the position of the 'i' frog is represented as:

$$P_i = (p_{i1}, p_{i2}, \dots, p_{iD})$$

Afterwards the performance of each frog is computed based on its position. The frogs are sorted in a descending order according to their fitness. Then, the entire population is divided into m memeplexes, each containing n frogs (i.e.,  $q = m \times n$ ). In this process, the first frog goes to the first memeplex, the second frog goes to the second memeplex, frog m goes to the mth memeplex, and frog m+1 goes to the first memeplex, and so on. Within each local memeplex, the frogs with the best and the worst fitness are identified as  $P_b$

and  $P_w$  respectively. Also, the frog with the global best fitness is identified as  $P_g$ . Then, an evolutionary process is applied to improve only the frog with the worst fitness (not all frogs) in each cycle. Accordingly, each frog updates its position to catch up with the best frog as follows:

$$Si = \text{Rand}() * (P_b - P_{w\text{Current}}) \quad (14)$$

$$P_{w\text{New}} = P_{w\text{Current}} + Si - S_{\text{max}} < Si < S_{\text{max}} \quad (15)$$

Where  $\text{Rand}()$  is a random number in the range  $[0, 1]$ , and  $S_{\text{max}}$  is the maximum step size allowed to be adopted by a frog after being infected. If this process produces a better solution, it replaces the worst frog. Otherwise, the calculations in Equations (14) and (15) are repeated with respect to the global best frog (i.e.,  $P_g$  replaces  $P_b$ ). If no improvement becomes possible in this case, then a new solution is randomly generated to replace the worst frog. The calculations then continue for a specific number of iterations [7].

## VII APPLICATION OF SHUFFLED FROG LEAPING FOR CT DESIGN AND CHROMOSOME STRUCTURE

The chromosome is defined as an array of random variables as follows:

$$P = [I_{mn} \ R_2 \ R_b \ X_l \ L] \quad (16)$$

$$X_t = X_2 + X_b \quad (17)$$

Where

$I_{mn}$ : rated magnetizing current in per-unit

$R_2$ : secondary winding resistance in per-unit

$R_b$ : burden resistance in per-unit

$X_2$ : leakage reactance in per-unit

$X_b$ : burden reactance in per-unit

$L$ : core length in meters

With respect to distributed winding, secondary leakage reactance is negligible. By several runs of algorithm, the highest convergence speed has been achieved with 20 numbers of populations in 5000 iteration.

In this paper for designing of CT, the core with type F which has the most permeability has been used. Technical data of designed transformers are mentioned in Table II. Performance of the algorithm for transformers of Table II has resulted design parameters of Table III. With respect to the equivalent circuit parameters resulted from designing, ratio and phase displacement errors and construction characteristics of design have been calculated and presented in Tables IV, V and VI respectively.

TABLE II. RATED VALUE OF DESIGNED CURRENT TRANSFORMER

CT	Ratio( $I_{1n} / I_{2n}$ )	S(V.A)
CT1	200/5	2.5
CT2	200/5	5
CT3	300/5	2.5
CT4	300/5	5

TABLE III. EQUIVALENT CIRCUIT PARAMETER AND BURDEN OF CURRENT TRANSFORMER RESULTED FROM DESIGNING

CT	$i_{mn}$	$R_2$	$R_b$	$X_t$
CT1	0.0045	0.1994	0.1732	0.1397
CT2	0.004	0.1038	0.0701	0.0548
CT3	0.0041	0.3281	0.3863	0.318
CT4	0.0037	0.1585	0.1563	0.1244

TABLE IV. RATIO ERROR VALUES IN DIFFERENT COEFFICIENTS FOF RATED CURRENT FOR DESIGNED CURRENT TRANSFORMER

CT	RATIO ERROR (%)				
	$0.25 I_{1n}$	$0.5 I_{1n}$	$0.75 I_{1n}$	$I_{1n}$	$1.2 I_{1n}$
CT1	-0.1563	-0.1568	-0.1574	-0.158	-0.1585
CT2	-0.1209	-0.1213	-0.1217	-0.1221	-0.1224
CT3	-0.1647	-0.1655	-0.1663	-0.1673	-0.1682
CT4	-0.1342	-0.1347	-0.1354	-0.1360	-0.1366

TABLE V. PHASE DISPLACEMENT ERROR VALUES IN DIFFERENT COEFFICIENTS OF RATED CURRENT FOR DESIGNED CURRENT TRANSFORMERS

CT	PHASE DISPLACEMENT ERROR (min)				
	$0.25 I_{1n}$	$0.5 I_{1n}$	$0.75 I_{1n}$	$I_{1n}$	$1.2 I_{1n}$
CT1	14.237	14.285	14.336	14.392	14.440
CT2	13.098	13.137	13.179	13.225	13.263
CT3	12.658	12.719	12.786	12.860	12.924
CT4	11.621	11.670	11.724	11.784	11.835

TABLE VI. CONSTRUCTION CHARACTERISTICS OF DESIGNED CURRENT TRANSFORMERS

CT	price(\$)	$l_{\text{core}}$ (m)	$A_{\text{core}}$ (cm <sup>2</sup> )	$l_{\text{wire}}$ (m)	$A_{\text{wire}}$ (mm <sup>2</sup> )
CT1	3.4435	0.2	8	3.310	3.321
CT2	3.4674	0.2	8	3.334	3.2134
CT3	3.5402	0.3	5	4.863	2.965
CT4	3.4276	0.3	6	4.765	3.0073

By evaluating of Tables IV, V it is clear that in designed current transformers, ratio and phase displacement errors are small so that ratio error is less than 0.17%, while the standard value for ratio error is 0.5%, also phase displacement error is less than 15 minutes, while the standard value for phase displacement error is 30 minutes. Table VI shows values that have been resulted from designing for CT dimensions. It should be mentioned that these dimensions has been attained with respect to practical constraints in construction of current transformer.

## VII. RESULTS

- CT1 and CT2 are smaller in cross section of conductor than CT3 and CT4 respectively (see Table VI) because CT1 has smaller primary current than CT3 but they have a same power and this reason can be applied about CT2 and CT4 (see Table II).

- CT2 is more expensive than CT1 because they have a same primary current but the power of CT2 is larger than the power of CT1. Also this expression can be used about CT3 and CT4 similarly (see Table VI).

- For some practical constraints, the length of core is large in CTs with large primary currents. For example the length of

core in CT3 and CT4 is larger than this value for CT1 and CT2 (see Table VI). It can be seen in Table VI that a transformer with are primary current (large ratio) has larger length of wire than another transformer with same power. For example the length of wire in CT3 and CT4 is larger than this value in CT1 and CT2.

#### CONCLUSION

In this paper a new approach based on Shuffled Frog Leaping Algorithm has been presented to design measurement-type current transformer. This method by using core data and other characteristics of CT has better performance in comparison with common methods of CT designing that are mainly based on trial and error. Advantages of this method are presenting of equivalent circuit parameters and magnetizing current and other parameters of design. In this approach, effects of burden change have been taken into account and finally, the rated burden of transformer to reach the expected ensuring-accuracy has been obtained.

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